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On the METHOD *of* DETERMINING *the* LONGITUDE, *by* OBSERVATIONS *of the* MERIDIAN PASSAGES of the MOON *and a* STAR *made at two Places. By the Rev.* JAMES ARCHIBALD HAMILTON, *D.D. Profeſſor of Aſtronomy at Armagh.*

Read Nov. 1, 1794.

General Rule for determining the eſtimate difference of the Longitude of a given place, from ſome well known meridian, by obſervations of the different intervals of A. R. of the moon's enlightened limb from a ſtar at each place on the ſame day.

LET the paſſage of the moon's enlightened limb be carefully obſerved to the neareſt tenth of a ſecond, over the ſeveral wires of the tranſit inſtrument, and let its paſſage over the centre be deduced by taking a mean of its paſſages over the equidiſtant wires, properly reduced. Let one or more fixed ſtars that lie near her parallel of declination, and alſo as near to her in A. R. as can conveniently be had, be obſerved in like manner. Let the rate of the ſydereal clock, which is the moſt convenient for this purpoſe, be carefully aſcertained, and from this and the obſervations collect the exact difference of the moon's A. R. from one or more of theſe ſtars under your own meridian. Having obtained correſponding obſervations from ſome ſettled obſervatory, find the difference of the differences of A. R. of the moon's limb from the ſame ſtar, under each meridian; take from the ephemeris her increaſe of A. R. for the 12H. including the obſervations, and ſay: As this quantity of A. R : 12H : : the apparent difference of the obſerved differences of A. R. to the eſtimate difference of longitude. This eſtimate difference will always be exact enough to enable you from tables or by calculation to find the corrections to which the obſervations and the lunar rate muſt neceſſarily be ſubjected, to deduce from them the true difference of longitude between the reſpective places of obſervation. To elucidate the nature and grounds of theſe ſeveral corrections, I ſhall premiſe ſome general conſiderations on the obſervation of the difference of A. R. of the moon's enlightened limb from a ſtar, and of the equation o the moon's increaſe of A. R. in 12H. as given in the ephemeris.

As this is acknowledged to be one of the moſt certain and convenient obſervations for determining the longitudes of places, the reduction that is required to give it its due accuracy ſeems deſerving of a more diffuſive explanation than as far as I can find has been hitherto beſtowed on it by any of the writers on practical aſtronomy. Mr. Vince, in his late treatiſe, mentions this method, as having been recommended by Dr. Maſkelyne in the year 1769; and reports the ſucceſsful application of it by Mr. E. Pigott, then of York, in deducing the difference of longitude of his obſervatory from that of Greenwich. Both he and Mr. E. Pigott, to whom indeed we principally owe the advantages that may, and moſt probably will accrue to geography from this method, beſides ſtating the general proportion, mention, that when the moon's rate is irregular attention ſhould be paid to that circumſtance; and Mr. Vince further adds, that the accuracy of the reſult will be greater, if we allow for the difference of the increaſe of the moon's A. R. in the intervals of the paſſages of the two meridians over the moon and ſtar, for that the obſerved difference is not accurately the true difference, when the two meridians paſs the ſtar.

As the fixed ſtar is the permanent and immutable term of the compariſon for both meridians, Mr. Vince very juſtly refers the difference of A. R. that is to be determined to the actual moment under each meridian, that theſe meridians apply to the ſtar and not to the moon's enlightened limb. The queſtion then, in its moſt ſimple and accurate form is, when the meridians ſucceſſively apply to the ſame ſtar, what is *then* the preciſe difference of the A. R. of the moon's enlightened limb from that of the ſtar for each meridian?

To elucidate this point—Let us ſuppoſe the paſſages to be obſerved by a clock, moving ſydereal time, and the meridian applies firſt to the moon's enlightened limb; its A. R. is then denoted by the clock. The ſame meridian preſently arrives at the ſtar, whoſe A. R. is in like manner expreſſed by the clock. The time elapſed between the obſervations would be the true interval of their A. R's. ſuppoſing the moon to have ſtood ſtill in her orbit—But the moon was increaſing her A. R. during the whole of the elapſed time; and of courſe every moment approaching nearer to the ſtar, therefore when the ſtar paſſed their difference of A. R. was decreaſed by the quantity of A. R. gained by the moon during the interval of ſydereal time elapſed ſince her own paſſage. If then a given meridian ſhould paſs the moon at 12H. and a ſtar at 14Hs. their apparent difference of A. R. would be 2H. and their true difference of A. R. at the moment of the paſſage of the ſtar would be the apparent difference—the moon's increaſe of A. R. in 2H. If under another meridian the moon ſhould paſs at 12H. 1′, and the ſtar at 14H. then their apparent difference of A. R. under that meridian would be 1H. 59′,—and their true difference A. R. at the time of the ſtar's paſſage 1H. 59′—the moon's increaſe of A. R. in 1H. 59′. Therefore the actual difference of theſe obſerved differences of A. R. is not the true quantity required, but muſt be leſſened by a quantity of A. R. proportioned to the moon's rate in her orbit; and the difference of the two ſydereal intervals of 2H. and 1H. 59′.; for let us ſuppoſe the moon to increaſe her A. R. at the regular rate of 30′ of a degree for an hour, and a meridian to paſs her firſt limb at 12H. and that the ſame meridian applies to a ſtar at 14H. at the time therefore of the ſtar's paſſage, the moon had ſince her own paſſage approached the ſtar by 60′ of motion = 4′ in time ∴ 2 H.—4′ or 1H. 56′ was the true interval of A. R. between theſe heavenly bodies when the ſtar paſſed. Let us again ſuppoſe the moon to move at the ſame rate in her orbit, and another meridian to paſs over her firſt limb when it had 12H. 1′ 0″ of A. R. The ſtar ſhe was before compared with being a fixed point, will be found by this ſecond meridian at its permament ſtation of 14H. and to get the true difference of the moon's A. R. from the ſaid ſtar at the inſtant of its paſſage 1H. 59′ the obſerved difference muſt be leſſened by the moon's increaſe of A. R. in 1H. 59′=to 59′. 30″ of motion, or 3′ and 58″ of time.

EXAMPLE.

Firſt Meridian.	H.	′	″
☽ 1 limb paſſes at - -	12	0	8
Star paſſes at - - -	14	0	0
Apparent difference of their A. R's. -	2	0	0
Deduct the moon's increaſe of A. R. in 2Hs.	0	4	0
True difference of their A. R's. when the ſtar paſſed	1	56	0

Second Meridian.	H.	′	″
☽ 1 limb paſſed at - -	12	1	0
Star paſſed at - -	14	0	0
Apparent difference of A. R's. -	1	59	0
Deduct for moon's increaſe of A. R. in 1H. 59′ 0″ - -		3	58
True difference of their A. R's. when the ſtar paſſed	1	55	2
Apparent or obſerved difference of A. R. at 1ſt meridian	2	0	0
Apparent or obſerved difference of A. R. at 2d ditto	1	59	0
Apparent difference of ☽'s A. R. for the interval of the meridians - - - -	0	1	0
True difference of A. R's when the ſtar paſſed 1ſt meridian	1	56	0
True difference of A. R's when the ſtar paſſed 2d meridian	1	55	2
True difference of ☽'s A. R. for the interval of the meridians - - - - -	0	0	58

Let us now take an example, at the ſame lunar rates and intervals of meridians as before, only ſuppoſing the ſtar to precede the moon.

Firſt Meridian.

	H.	′	″
Star paſſes at - -	12	0	0
☽'s 1 limb paſſes at -	14	0	0
Apparent or obſerved difference of their A. R's	2	0	0
Deduct for moon's increaſe of A. R. ſince the ſtar paſſed	0	4	0
True difference of their A. R's when the ſtar paſſed -	1	56	0

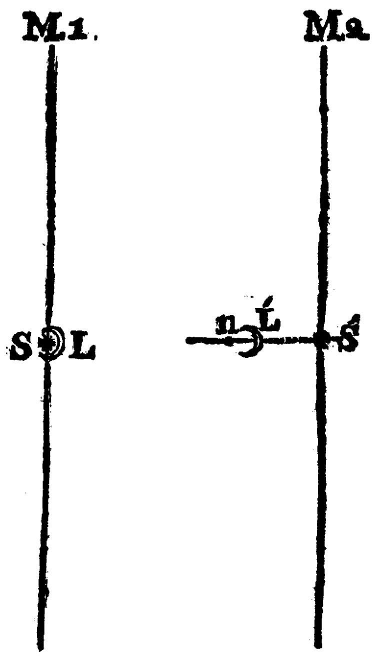
Second Meridian.

Star paſſes at - -	12	0	0
☽ 1 limb paſſes at - -	14	1	0
Apparent or obſerved difference of their A. R's.	2	1	0
Deduct for ☽'s increaſe A. R. in 2H. 1′ ſince the ſtar paſſed	0	4	2
True difference of their A. R's when the ſtar paſſed -	1	56	58

Apparent or obſerved difference of A. R's at 1ſt meridian	2	0	0
Apparent or obſerved difference of A. R's at 2d meridian	2	1	0
Apparent change of ☽'s A. R. for the interval of meridians	0	1	0
True difference of A. R. when the ſtar paſſed 1ſt meridian - - - - - -	1	56	0
True difference of A. R. when the ſtar paſſed 2d do.	1	56	58
True change of ☽'s A. R. for the interval of meridians	0	0	58

HENCE it is evident, that in each caſe the obſerved difference of the differences of A. R. is to be leſſened by a quantity proportioned to the moon's ſuppoſed uniform rate, and the interval of the meridians; the moon's rate therefore being given, the abſolute quantity of the correction will depend on the diſtance of the meridians aſunder.

NOTE—The whole of the ſydereal interval between the paſſages of the moon's enlightened limb and the ſtar, being common to both places of obſervation, 1′ only excepted, it is for this minute only which is the difference of the ſydereal interval between the moon and ſtar at the ſeparate places of obſervation, ariſing from the moon's motion in her orbit during the interval of the paſſages of the two meridians, that the effective correction, amounting to 2″ in this inſtance, takes place.



The ſimpleſt form that can be deviſed for explaining the nature of this correction, is to ſuppoſe, as in the annexed figure, that the moon's enlightened limb L′, and a ſtar S on her parallel, apply at the ſame inſtant to 1ſt meridian M. 1. then the true and obſerved difference of A. R. will there be the ſame, viz. o. then let the ſtar S′ apply to a ſecond meridian M. 2. 3 H. diſtant from the former; and let the moon changing her A. R. at the uniform rate of 24′ time in 12 hours be arrived at the point L′, having receded from the ſtar by 6′ of A. R. in time, when the ſtar S′ is on the 2d meridian, and the interval L′ S′ the difference of A. R. which compared with the moon's increaſe of A. R. in 12H. will give the difference of meridian or of longitude. But while the point L is coming to the 2d meridian the moon will have receded to the point ṇ, and when her enlightened limb arrives at the 2d meridian the obſerved difference of A. R. between the ſtar and it will be n S′ and the correction of this obſerved difference will be ṇ L′, it is obvious this quantity will increaſe with the quantity L′ S′, and the quantity L′ S′ will be in the direct proportion of the increaſing interval of 1ſt and 2d meridian, and of the moon's actual rate in her orbit.

If the moon is not compared with any particular ſtar at either meridian, but her apparent A. R. on the meridian obſerved at each place by well regulated ſydereal clocks — The ſame correction of the obſerved difference of A. R. is required, as though her enlightened limb was compared with a given fixed ſtar; for in this caſe ſhe is, in fact, compared with an imaginary ſtar in the firſt of aries, her obſerved diſtances from which are ſubject to the ſame regulations as though there was a real ſtar at that point in the heavens.

It may be farther neceſſary to add on this ſubject, that if at the different places of obſervation the moon ſhould be on the oppoſite ſides of the ſtar with which ſhe is compared, then the ſum of the intervals of her A. R. from that of the ſtar is to be uſed in this calculation inſtead of the difference of theſe intervals.

When the meridians are even a moderate diſtance from each other, the neglect of the equation for the difference of the obſerved and true interval will have a very conſiderable influence on the reſult of the obſervations; when the moon moves at her quickeſt rate, and the meridians are only 10° aſunder, the correction amounts to 5″ of time on the obſerved interval, it will of courſe in all caſes require to be very ſtrictly attended to. To facilitate therefore this part of the operation, I have calculated a ſhort table, intitled " A table of corrections, &c." for which the correction of the interval for any lunar rate, and for any longitude not exceeding 10° may be taken out with ſufficient accuracy almoſt at ſight. The correction in this table is given in time to ſeconds, tenths, &c. that it may be directly applied to the difference of the A. R's. as found immediately by the tranſit inſtrument. The difference thus corrected is to be changed into parts of a degree, to prepare it for the analogy from whence the longitude is to be finally deduced.

We have hitherto conſidered the moon's rate in her orbit to be equable, but as this is ſeldom in reality the caſe, this method requires that when her rate is unequal, the ſum of her A. R. gained in 12H. ſhould be equated as nearly as poſſible,

according

according to the rate ſhe moved at, at the time of the mid-interval of the obſervations.

To explain this, let us ſuppoſe the moon, by an equable motion, to have changed her A. R. in 12H. by 6°, and on that day under a different meridian her difference of A. R. from the ſame ſtar with which ſhe had been compared at a former meridian was found to be encreaſed by 1°, then 6°: 12H. : : 1° : 2H. the longitude required. Now on another day let us ſuppoſe the moon to have gained as before 6° A. R. in 12H. but at the following rates for each hour of the 12H.

H.		H.		H.		H.	
1ſt	30′. 6″	4	30′. 3″	7	30′. 0″	10	29′. 56″
2	30. 5	5	30. 2	8	29. 58	11	29. 55
3	30. 4	6	30. 0	9	29. 57	12	29. 54

Should the moon's A. R. be compared with the ſame ſtar at the ſame places on this day at the beginning of the 12 hours ſhe will be found to have actually gained in the interval of the paſſages of the 2 meridians 1°. 0′. 11″. and the proportion unequated would be 6° : 12H. : : 1°. 0′. 11″ : 2H. 0′. 22″. and ſhould the obſervations happen near the end of the 12H. the moon would be found to have gained in the ſame interval only 59′. 49″. of A. R. and the unequated proportion would run thus, 6° : 12H. : : 59′. 49″ : 1H. 59. 38; whence it is evident that the longitude deduced from theſe obſervations would be in one caſe 22″ too great, and in the other 22″ leſs than the truth: This ſource of error therefore ſhould be corrected before the proportion is ſtated, by altering the firſt term in conformity to the moon's actual rate during the interval between the obſervations. Thus in the firſt analogy the deviation in 2 hours from the mean rate of 30″ per hour is + 11″; this multiplied by 6 gives 1′ 6″, which added according to its ſign to the firſt term of the proportion would make it 6°. 1′. 6″, and the proportion would ſtand thus, 6°. 1. 6 : 12H : : 1. 0. 11″ : 2H. the ground and neceſſity of this correction are evident, and when it is not applied the irregularity of the moon's rate being given, the quantity of error will be greater or leſs in proportion as the obſervations were made at a part of the 12H. more or leſs diſtant from the hour of the mean rate. Had theſe ſuppoſed obſervations been made between the 5th or 7th hours the longitude would have come out true without any correction of the rate whatſoever.

Having premiſed thus much on the nature and grounds of this correction, I ſhall proceed to a ſhort explanation of the readieſt method of making it under every poſſible change of of the moon's rate with due accuracy and preciſion.

M. de la Lande, in his Aſtronomy, liv. 7. article 1521, obſerves that when the longitudes or A. R's of the moon are ſtrictly calculated from 12H. to 12H. as they are given in the Nautical Almanacs and other Ephemerides, you may thence deduce the horary motion for any part of this period to a great degree of preciſion; for if you take the 12th part of the moon's movement in longitude or A. R. for 12H. from noon to midnight, or from midnight to the ſucceeding noon, you will have the horary motion either in longitude or A. R. that took place at 6H. or 18H. For by the method of interpolation, vide article 3928, he aſcertained that in ſpite of all the irregularities of the lunar motions the 2d differences proceed at a rate that is to all ſenſe uniform even for the ſpace of 24 hours; it is evident therefore, that having obtained the horary motion, for 6 or 18Hs. by taking the $\frac{1}{12}$ of the movement for 12H. you may then have it for any other hour of that interval by an even proportion.

As the A. R's. are the only lunar motions we have occaſion to make uſe of, in this method for the longitude of places, I ſhall only conſider the manner of finding the moon's horary motion in A. R. for any required minute of any interval of 12, and thence of aſcertaining the exact quantity of the A. R. for 12H. that is to be uſed both for the correction of the interval of obſerved A. R. between the 2 meridians, and alſo for the correction of the eſtimate longitude.

Take out of the Nautical Almanac, or any other good Ephemeris, the two calculated A. R's. immediately preceding, and the two that immediately follow, the hour and minute for which you want to determine the moon's horary motion; place theſe in four ſucceſſive lines below each other, and by their ſides their three ſucceſſive differences for each 12 hours; multiply each of theſe laſt numbers by 5, taking the degrees for minutes and the minutes for ſeconds, and you will have three other numbers that will accurately denote the horary motions at the mid-intervals of the pairs they reſpectively belong to. The reaſon of multiplying by 5, &c. is merely for brevity and convenience in finding the $\frac{1}{12}$ of the motion in A. R. for 12H. for $a \times 60' \div 12 = a \times 5'$, which laſt is much the moſt conveniently and readily done.

Having thus got the horary motions for the mid-intervals of 6H. and 18H. to get them for any other hour and minute you have only to take the difference of the horary motions at the beginning and end of the period of 12Hs. that includes the hour and minute for which the rate is required, and ſay, as 12H. is to the whole difference, ſo is any other number of hours and minutes to a proportionate part; which added or ſubtracted as occaſion requires, to or from the rate of the neareſt mid-interval, gives the rate at the time demanded. Let it be required, for example, to find the moon's horary motion in A. R. for 6H. 30′. on the 5th Jan. 1789.

				× by 5					H.
Jan. 4, at	12	15°	51′	6°	4′	30′	20″	horary motion 4th day at	18
5, at	0	21	55	6	16	31	20	horary motion 5th day at	6
5, at	12	28	11	6	28	32	20	horary motion 5th day at	18
6, at	0	34	39						

Here 30′ being the exceſs of the time for which the rate is required above the laſt mid-interval, and 1′ the variation of the horary motion for 12H. ſay, 12 : 1 : : 30′ : 2″ 30‴, which added to 31′. 20″ the rate of the horary motion at 6H. gives the horary motion at 6H. 30′ = to 31′. 22″. 30‴. If we now ſuppoſe the mid-interval of the moon's paſſage over two ſucceſſive meridians, at each of which the diſtance of her enlightened limb from the ſame ſtar was obſerved to have happened at this very time, then to determine the difference of their longitudes from theſe obſervations her whole increaſe of A. R. for 12 hours, at the preciſe rate at which ſhe then moved, is required; and this quantity is had by multiplying the horary motion at 6H. 30′ by 12—thus, 31′. 22″. 30‴. × 12 = 6°. 16′. 30″.

All that ſeems now requiſite is to add a few words on the meaſure of time to be employed in the calculation of the longitude, by this method of comparing the ☽'s limb, when on two different meridians, with the ſame fixed ſtar.

As the calculations in the Nautical Almanac, of all the lunar motions, are expreſsly declared to be made for the apparent moments of noon and midnight, as deduced immediately from the ſun, and as this apparent time, about the 20th of December, varies 30″ in a day from M. ☉ time, it is evident that the change of the ☽'s A. R. for 12H. at that time, as given in the Nautical Almanac, is in fact, the change for 12H. 0′ 15″ of M. ☉ T. In this caſe therefore the A. R. gained in 15″. ſhould be deducted from the ſum of the increaſe of the ☽'s A. R. in 12H. apparent time; that the remainder may correſpond to the 2d term of the analogy (viz. 12H.) in M. ☉ T. and thus the 4th term of the proportion, namely the difference of longitudes, will alſo come out in this equable aſtronomic meaſure of time. This correction will take place wherever the diurnal difference between mean and apparent time is great enough to make any ſenſible difference in the firſt term of the proportion.

The reſult thus obtained, will however require a ſmall correction in the amount of the longitude, as found by this lunar problem.

For as 24H. M. ☉ T. amount to 360. 59. 8 ∓, and as the whole æquator comprizes but 360°. 1H. of M. ☉ T. ſhould indicate 15°. 2'. 8". of the æquator, and as terreſtrial longitudes are counted at the rate of only 15° to an hour, the longitudes given in M. ☉ T. ſhould be corrected according to this difference, and the quantity of this correction will be readily ſhewn by the tables for converting M. ſolar into ſydereal time.

Or perhaps it may be as ready a way to reduce the 12H. of M. ſolar time into ſydereal before the proportion is made, and thus the three firſt terms in the proportion being in this meaſure of time, the fourth will conſequently come out in the ſame, and no further correction be required.

Take an Example each way:

1ſt. 8° : 12 H. M ☉ T. : : 40′ : 1 H. M. ☉ T. : but 1 H. M. ☉ T. = 1 H. 0′. 9″, 83 ſydereal time.

2d. 12 H. M. ☉ T. = 12 H. 1′. 58″, 283 ſydereal time.

Therefore 8° : 12 H. 1′. 58″, 283 : : 40′ : 1 H. 0′ 9″, 83.

It is further to be obſerved that though the two corrections I have juſt ſtated are the principal ones that uſually occur in this calculation, and indeed the only ones that for meridians of ſhort intervals aſunder are ſenſibly requiſite, yet in order to diſcuſs this queſtion fully it may be neceſſary to mention ſome other ſources of irregularity, that at particular times and at places conſiderably diſtant from each other in longitude, well deſerve the notice of the attentive and ſcrupulous aſtronomer. The apparent diſk of the moon (as ſeen from the centre of the earth) being a quantity perpetually varying in its dimenſions, if its change ſhould be ſenſible in the interval between the obſervations at the firſt and ſecond meridian, it is evident that in proportion as the diameter of the moon is either contracted or expanded during this interval, the enlightened limb which ſhould be a fixed boundary of menſuration for each place will either retire farther from or be protruded towards the ſtar more under the ſecond meridian than it was under the former; and thus, independent of all other cauſes of error, vitiate the accuracy of the meaſurement. This alteration of the diſk happens from the moon's changing her abſolute diſtance from the earth during the interval of the obſervations. As the greateſt variation of the moon's ſemidiameter is about 16″ in a day, in this extreme caſe, under meridians that are 6H. aſunder, the correction of the diſk from this cauſe only would amount to 4″. And ſhould ſhe in the interval have changed her declination, ſo as to vary her ſemidiameter by another ſecond in the ſame direction with the former variation, then the aggregate of theſe alterations of the lunar diſk would collectively amount to five ſeconds, a quantity that would make a conſiderable error in the determination of the longitude.

The neceſſary corrections being found from the eſtimate longitude and the tables, and carefully applied, all that remains to be done is to ſtate over again the analogy mentioned in the general rule, ſubſtituting therein the corrected numbers. To render this final analogy as brief and as eaſy as poſſible, I have added a ſecond table of *aſſiſtant logarithms* for every minute of the moon's increaſe of A. R. for 12H. in which the arithmetical complement of the proportional logarithm of the quantity of the moon's increaſe of A. R. in 12H. for every change of 1′ in her rate is added to the proportional logarithm of 12H. ſo that all that is neceſſary is to change the corrected difference in time into parts of a degree, and to add its proportional logarithm to the logarithm taken out of the table, and the ſum will be the proportional logarithm of the difference of the two meridians in time.

EXAMPLE.

JANUARY 5th, 1789. The meridians of Greenwich, and of the obſervatory of Trinity College, Dublin, (eſtimated longitude 25′. 18″. weſt of Greenwich) were obſerved to apply ſucceſſively to α Aquilæ and the ☽'s firſt limb at the following intervals of A. R. in ſydereal time.

GREENWICH.		OBSERVATORY, T. C. D.	
α Aquilæ	19H. 40′. 32″. 15	α Aquilæ	19H. 40′. 52″, 8.
☽'s 1ſt limb	1. 40. 8. 50	☽'s 1ſt limb	1. 41. 26, 3.

Apparent difference of A. R. at Greenwich 5H. 59′ 36″, 35. Clock moving nearly ſydereal time.

Apparent difference of A. R. at Obſervatory, T. C. D.	6H. 0′. 33″, 5
Deduct for clock's rate	2, 6
	6. 0. 30, 9
Apparent difference of A. R. at Greenwich	5. 59. 36, 35
Appar. change of A. R. in the interval of meridians	0. 0. 54, 55
Correction from Table I.	1, 90
True change of A. R. in the interval	52, 65

Change of ☽'s A. R. for 12 hours at the rate of 6H. 33,
being the mid-interval of the obſervation 6°. 16′. 30
Its aſſiſtant logarithm from Table II. 9. 7185
prop. logarithm of 52″, 65 = 13′. 9″, 7 1. 1360

0. 8545

prop. logarithm of 25′ 9″, 6, which reduced to ſydereal time is 25′. 13″, 7 longitude weſt.

NOTE.—Theſe appulſes of the moon and ſtar to the meridians of Greenwich and of the Obſervatory of T. C. D. were obſerved with the excellent tranſit inſtruments that thoſe obſervatories are furniſhed with—and reduced by a mean of the five wires in the reſpective tranſit books of Drs. Maſkelyne and Uſsher.

On the Conſtruction of Table I.

As the correction of the difference of the diſtances of the moon's enlightened limb, from the ſame ſtar under different meridians, evidently depends on the interval of the meridians, combined with the moon's rate at the mid-interval of the obſervations, the table intended to facilitate this correction was conſtructed in the following manner :

LET S be the ſum of the moon's increaſe in A. R. for 12H. H 12 H. L the eſtimate longitude, D the difference of

A. R. correſponding to the moon's rate and the eſtimate longitude, and C the correction required.

Then H : S : : L+D : D & H : S : : D : C.
Or H-S : S :: L : D.
Conſequently $D=\frac{S\times L}{H-S}$ or $D=\frac{S\times L}{H}\times\frac{1}{1-\frac{S}{H}}=\frac{SL}{H}\times\overline{1+\frac{S}{H}+}$, &c.

& $\frac{D\times S}{H}=C$.

TABLE I.

Corrections of the apparent differences of the diſtances of the moon's limb from a ſtar, as obſerved on the ſame day under different meridians, in parts of ſydereal time.

Diff.	Long.	Increaſe of the Moon's A. R. in 12H.					
Space. 0	Time. 1	5°.34	6° 4′	6°. 34′	7°. 4′	7°. 34′	8°. 4′
1	4	0, 24	0. 28	0, 33	0, 38	0, 43	0, 49
2	8	0, 47	0, 57	0, 66	0, 76	0, 86	0, 97
3	12	0, 71	0, 85	0, 99	1, 14	1, 30	1, 47
4	16	0, 96	1, 14	1, 32	1, 53	1, 73	1, 96
5	20	1, 20	1, 42	1, 65	1, 91	2, 17	2, 46
6	24	1, 45	1. 70	1 98	2, 29	2, 60	2, 94
7	28	1, 68	1, 98	2, 31	2, 67	3, 04	3, 43
8	32	1, 92	2, 27	2, 64	3. 06	3, 47	3, 93
9	36	2, 15	2, 55	2, 97	3 44	3. 90	4, 42
10	40	2, 38	2, 84	3, 31	3, 82	4. 34	4, 94

This table is to be entered at top with the moon's ſuppoſed equable rate for 12 hours aſſumed from her true rate, at the mid-interval of her paſſage from the meridian of the firſt place of obſervation to that of the ſecond; and at the ſide with the eſtimate difference of the longitudes of the places in time or motion.

N. B. In conſtructing the above table, the two firſt terms of the ſeries $\frac{S\,L}{H} \times \overline{1 + \frac{S}{H} +, \&c.}$ were only uſed as giving a reſult ſufficiently accurate.

A Table of aſſiſtant logarithms to facilitate the proportion that is to be made for deducing the longitude from the corrected differences of the A. R's. of the moon's enlightened limb from that of a ſtar, obſerved on the ſame day under different meridians, being

TABLE II.

′	5°	′	5°	′	6°	′	6°	′	6°
30	9.6612	50	9.6868	9	9.7097	29	9.7326	49	9.7544
31	6625	51	6880	10	7109	30	7337	50	7555
32	6638	52	6892	11	7121	31	7349	51	7565
33	6651	53	6905	12	7132	32	7360	52	7576
34	6664	54	6917	13	7144	33	7371	53	7586
35	6677	55	6929	14	7156	34	7382	54	7596
36	6690	56	6941	15	7167	35	7393	55	7607
37	6703	57	6953	16	7179	36	7404	56	7618
38	6716	58	6966	17	7190	37	7415	57	7628
39	6729	59	6978	18	7202	38	7426	58	7639
40	6742		6°	19	7213	39	7436	59	7649
41	6755	0	9.6990	20	7225	40	7448		7°
42	6767	1	7002	21	7236	41	7458	0	9.7659
43	6780	2	7014	22	7248	42	7469	1	7670
44	6793	3	7026	23	7259	43	7480	2	7680
45	6805	4	7038	24	7270	44	7491	3	7690
46	6818	5	7050	25	7282	45	7501	4	7701
47	6830	6	7062	26	7293	46	7512	5	7711
48	6843	7	7074	27	7304	47	7523	6	7721
49	6855	8	7085	28	7315	48	7533	7	7731

TABLE II. *continued.*

′	7°	′	7°	′	7°	′	8°
8	9. 7741	28	9. 7940	48	9. 8129	7	9. 8302
9	7751	29	7949	49	8139	8	8311
10	7762	30	7959	50	8148	9	8320
11	7772	31	7969	51	8157	10	8329
12	7782	32	7978	52	8166	11	8338
13	7792	33	7988	53	8176	12	8346
14	7802	34	7998	54	8185	13	8355
15	7812	35	8007	55	8194	14	8364
16	7822	36	8016	56	8203	15	8373
17	7832	37	8026	57	8212	16	8382
18	7842	38	8036	58	8221	17	8391
19	7852	39	8045	59	8230	18	8399
20	7862	40	8055		8°	19	8408
21	7871	41	8064	0	9. 8239	20	8417
22	7881	42	8073	1	8248	21	
23	7891	43	8083	2	8257	22	
24	7901	44	8092	3	8266	23	
25	7911	45	8101	4	8275	24	
26	7920	46	8111	5	8284	25	
27	7930	47	8120	6	8293	26	

The reſults of the following obſervations, made by Dr. Maſkelyne and Dr. Uſſher, at the obſervatories of Greenwich and Trinity College, Dublin, will tend to ſhew the advantages to be derived from this method.

		At Greenwich.		At the Obſervatory of Trinity College.		Difference of Longit. deduced.
		A. R. by the clock.	Clock d.Rate	A. R. by the clock.	Clock's d. Rate.	
1788	Procyon	7H 31′ 44″, 74	+0″, 5	7 28 43, 02	- 4″, 5	25′ 22″, 6 w
May 13	☽ 1 L	10 31 4, 25		10 26 57, 7		
June 16	Antares	16 16 16, 30		16 14 49, 9		25 5, 0
	☽ 1 L	16 20 10, 66		16 19 41, 4		
Aug. 17	α Aquilæ	19 40 41, 94	+ 0, 5	19 39 40, 05	- 8, 0	25 48, 9
	☽ 2 L *	22 32 12, 59		22 31 59, 28		
Oct. 7	☽ 1 L	19 31 40, 38		19 34 4, 3	- 13, 0	25 10, 4
	β Aquilæ	19 44 54, 38		19 46 25, 7		
1789	Aquilæ	19 40 32, 15	+ 0, 6	19 40 52, 8	+ 11, 0	25 13, 7
Jan. 5	☽ 1 L	1 40 8, 50		1 41 26, 3		
Jan. 8	Aldebaran	4 23 57, 12		4 25 21, 4	+ 10, 0	24 50, 2
	☽ 1 L	4 32 57, 90		4 35 27, 6		
Feb. 5	Aldebaran	4 23 44, 22	- 0, 8	4 24 38, 9	+ 15, 6	25 12, 0
	☽ 1 L	5 4 48, 82		5 6 49, 6		
Feb. 7	☽ 1 L	7 12 39, 80	- 0, 7	7 15 15, 7	+ 15, 0	25 16, 6
	Pollux	7 32 16, 68		7 33 44, 9		
Feb. 9	Procyon	7 28 7, 60	- 0, 4	7 30 6,	+ 14, 5	24 57, 5
	☽ 1 L	9 18 44, 98		9 21 48, 2		
Sept. 27	☽ 1 L	19 38 47, 00	- 0, 7	19 38 7, 1	- 20, 0	25 15, 5
	β Aquilæ	19 44 50, 42		19 43 16, 3		
Nov. 26	α Aquilæ	19 40 4, 92	+ 0, 3	19 39 0, 29	- 20, 0	25 3, 5
	☽ 1 L	23 46 40, 48		23 46 20, 3		
Dec. 24	☽ 1 L.	0 13 25, 72	- 0, 6	0 13 16, 9	- 17, 8	25 10, 0
	Aldebaran	4 23 15, 98		4 22 15, 8		
Dec. 25	☽ 1 L	7 0 4, 72	- 0, 6	0 59 38, 7	- 17, 8	25 9, 8
	Aldebaran	4 23 15, 06		4 21 57, 0		
Dec. 31	☽	6 27 31, 28	- 0, 6	6 27 33, 27	- 18, 0	24 50, 6
	Pollux	7 31 44, 08		7 30 42, 8		

☞ *For * ſee next Page.*

* This obſervation of the ſecond limb gives the difference of longitude greater than the obſervations of the preceding limb; which is to be attributed to the difference of the teleſcopes. — The Greenwich tranſit inſtrument gives a leſs diameter of the ſun than the tranſit inſtrument of the obſervatory of T. C. D. and the effect of this difference of inſtruments is to leſſen the longitudes deduced by the above method from obſervations of the firſt limb, and to increaſe thoſe deduced from the ſecond limb. Conſequently to uſe this method to moſt advantage a mean of reſults deduced from obſervations of both limbs ought to be taken.